

Introduction to Mathematical Modeling

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This lab follows the paper **Dynamical Models of Love** by J.C. Sprott, *University of Wisconsin, Madison*. The major change we make is to assume the model to be discrete.¹

SIMPLE LINEAR MODEL

In discrete form we can write a model for a love affair between Romeo and Juliet as:

$$R(n+1) = R(n) + aR(n) + bJ(n)$$

$$J(n+1) = J(n) + cR(n) + dJ(n)$$

Here R is the amount of love that Romeo feels for Juliet and J is the amount of love Juliet feels for Romeo. Assume that n is measured in days. This is like saying that they reassess their love for each other once a day. The constants a , b , c and d are parameters that can describe the type of people Romeo and Juliet are, aka their romantic style.

The parameter a describes the extent to which Romeo is encouraged by his own feelings, and b is the extent to which he is encouraged by Juliet's feelings. Similarly, the parameter d describes the extent to which Juliet is encouraged by her own feelings, and c is the extent to which she is encouraged by Romeo's feelings. Note that we assume that a negative value for love means hate, or fighting.

We can imagine four different Romantic Styles:

1. **Eager beaver:** the person is encouraged by their own feelings and by the other persons feelings. In terms of Romeo this would be $a > 0$ and $b > 0$.
2. **Narcissistic nerd:** The person wants more of what he feels but retreats from the other persons feelings. In terms of Romeo this would be $a > 0$ and $b < 0$.
3. **Cautious lover:** the person retreats from their own feelings but is encouraged by the other persons feelings. In terms of Romeo this would be $a < 0$ and $b > 0$.
4. **Hermit:** The person retreats from their own feelings and the feelings of others. In terms of Romeo this would be $a < 0$ and $b < 0$.

We can also have the case where the person either does not feel their own feelings, for Romeo $a = 0$, or the case where they have no regard for the other persons feelings, for Romeo $b = 0$.

Your goal is to investigate, using a spreadsheet, what the model predicts for different kinds of people. Here are some cases to start with, but feel free to think up your own.

- Assume they are out of touch with their own feelings: $a = d = 0$. What happens for different types of people. What if they are both eager beavers? What about two nerds? What about one eager beaver and one nerd? How do the initial conditions effect the solution? (AKA do first impressions count?)
- Assume the two lovers are exact opposites, fire and ice. Meaning $c = -b$ and $d = -a$. How do your outcomes change depending on whether the people respond more to themselves, for Romeo this is $|a| > |b|$, or more to the other person, for Romeo this is $|a| < |b|$.
- Peas in a pod, the two lovers are the same type: $c = b$ and $d = a$. Consider some of the different types. Do they end up in love or at war? How do first impressions effect the final solution.
- Romeo the robot, one person is totally unaffected by their surroundings, for Romeo this is $a = 0$ and $b = 0$. How does this effect the other persons feelings?

Feel free to come up with your own scenarios. Think of the outcomes of your model in terms of real people, does the model make good predictions?

¹The original idea for this model comes from a book by Steven Strogatz: *Nonlinear dynamics and chaos, With applications to physics, biology, chemistry, and engineering*, an awesome and readable book.

TO BE HANDED IN: I would like you to hand in a typed document describing what you found. First find fixed points for this system. What do the fixed points mean in terms of the "real life" system? Now program your spreadsheet and test the stability of the fixed points for different parameter values (aka different types of people). Test at minimum four different combinations of people and write a short paragraph about what you find. Do your model results make sense? Feel free to test as many combinations as you want. It is fun to think of real people or fictional characters who match the descriptions. Try to find at least one stable and one unstable case.

Want a challenge? Try modeling a love triangle. Now Romeo is interested in two women but the women are unaware of each other. Why does this model make sense? Draw the diagrams if you are not sure!

$$R_j(n+1) = R_j(n) + aR_j(n) + b(J(n) - G(n)) \quad (1)$$

$$J(n+1) = J(n) + cR_j(n) + dJ(n) \quad (2)$$

$$R_g(n+1) = R_g(n) + aR_g(n) + b(G(n) - J(n)) \quad (3)$$

$$G(n+1) = G(n) + eR_g(n) + fG(n) \quad (4)$$

where R_j represents Romeo's feelings toward Juliet, J represents Juliet's feelings toward Romeo, R_g represents Romeo's feelings toward Guinevere, and G represents Guinevere's feelings toward Romeo. Again the lower case letters are parameters describing how much each term affects the persons feelings.