Introduction to Mathematical Modeling

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This lab has two parts. **First**, you will collect data and statistically analyze that data. **Second**, you will build a stochastic simulation that models the data. This will allow you to compare the real life data and your simulated data.

PART 1 – DATA COLLECTION

We will begin by collecting some data. Each group should have a stack of 50 squirrels. Think of this as $P(0) = 50$. To simulate the death of squirrels we will be throwing the squirrels into the air. If they land face up they survived. If they land face down they did not survive and should be removed from the population. Each time you throw the squirrels in the air it simulates one year of their lives with a 50% survival rate. In our SHARED CLASS spreadsheet, enter your values going down the columns. Please "flip" your squirrels for $n = 10$ years. If the population reaches zero then all remaining years will be zero. This is ONE SIMULATION! It will help to have a few people as flippers and one person as data entry.

Repeat your squirrel killing simulations to get a decent amount of data to analyze. (Between 3-5 simulations is good since we are sharing data). Try to be quick so we all have time to finish the lab in class.

Once everyone is done with their simulations, copy and paste all of the data into YOUR OWN spreadsheet and try to answer the following questions:

- What was the average population each year?
- What was the standard deviation of population each year?
- What percent of the population, on average, did not survive each year?
- How do these values compare to the deterministic model that we did in class with 50 percent death rate?
- NOTE you can build the 50 percent survival rate model very easily: $P(n+1) = 0.5P(n)$ and calculate values. Then plot this line along with lines for each of your simulations.

SOME HELPFUL SPREADSHEET COMMANDS:

- **=average(B1:K1)** $\rightarrow$ finds the average of the values along the first row.
- **=stdev(B1:K1)** $\rightarrow$ finds the standard deviation of the row of values.
- **=median(B1:K1)** $\rightarrow$ finds the median of the row of values.
- **=quartile(B1:K1,1)** $\rightarrow$ finds the first quartile of the row of values.
- **=quartile(B1:K1,3)** $\rightarrow$ finds the third quartile of the row of values.
- **=rand()** $\rightarrow$ generates a random number between 0 and 1. (Uniformly Distributed)
- **=if(rand<0.5,0,1)** $\rightarrow$ generates a random number and tests if it is less than 0.5. Output of zero means less than 0.5 output of one means greater than 0.5.

PART 2 - STOCHASTIC MODEL BUILDING

Now we can try to build our first stochastic model. (NOTE my example spreadsheet is available on the class Website.) **Follow the directions carefully to build your first stochastic simulation.**

FIRST STEPS

- Open a spreadsheet and name the first page "Stochastic Model - Simulation"
- At the top of the spreadsheet name and enter your one and only parameter: Probability of survival $s = 0.5$, meaning the squirrels have a 50 percent chance of surviving each year. I put my parameter in cell C1.
- Now, we will build a table where you will collect the "simulation data". First make a column named YEAR starting in cell A6 with the numbers 0-10 below it and then make a second column named POPULATION starting in cell B6. In the first entry for population type 50, since we start with 50 animals. Leave the rest of the population column blank for now.

SIMULATION OF RANDOM EVENTS

- In cell A2 type "TO RERUN SIM CHANGE NUMBER:" and in cell B2 type any number.
- In cell E1 type the word SQUIRREL NUMBER. In cell F1 type YEAR 1, and cell G1 type YEAR 2, and so on until you have 10 years along the columns.
- In the column cells below SQUIRRELS, enter the numbers 1-50, to number the 50 squirrels that you start with alive.
- In cell F2 below the heading YEAR 1 we simulate our first year of survival for each squirrel. To do this we flip all of the squirrels by checking a random number. We need an IF statement that checks a random number and if that random number is less than your probability of survival, then the squirrel died. For my spreadsheet this looks like:

  =IF(RAND()<C1,0,1)

  Copy this down the column for all 50 squirrels. This will simulate the first set of 50 flips.
- In cell G2 below the heading YEAR 2 we simulate the next year. To do this we first need to check if the squirrel lived through the first year and then if it did survive we flip it again, check a random number. For my spreadsheet this looks like:

  =IF(F2=0,0,IF(RAND()<C1,0,1))

  Notice how there are two IF statements here. Copy this down the column for all 50 squirrels. This will simulate the second flip. If the first flip resulted in a zero then the second results remains zero, since we remove the dead squirrels, otherwise they have a 50% chance of survival.
- Drag cell G2 across for the other YEARS and then down for all 50 of the squirrels. This should fill out a table of zeros and ones.
- At the bottom of the zeros and ones table, we need to count the number of survivors each year. Do this by summing the cells above. For example in cells F52 type = sum(F2:F51) and so on to get the population count each year. Copy this across all ten years.

Your simulation is complete. Now in your small table of values, cells A6-B17, where you have a column named POPULATION tell those cells to look at the bottom of the simulation to see what the population is at each time step. For my spreadsheet for year 1 this looks like:

=F52

This gives you ONE SIMULATION.

MULTIPLE SIMULATIONS - MODEL OUTPUT

Go to a new sheet and call it "Model Data Collection". To refresh the random numbers (aka do a new simulation) type any new number into the Model spreadsheet cell B2. This should change the numbers in the Population table! Anytime you change a number in the spreadsheet it reevaluates all the random numbers. Copy the data in the POPULATION column and **PASTE SPECIAL** VALUES ONLY into the Model Data Collection sheet, this will record separate simulations. Do this at least 12 times for 12 different simulations.

- Find the average and standard deviation of the population each year.
- How does this compare to your real life "squirrel flips"?
- Plot the average values of your simulation using the standard deviation as error bars.