

Nonlinear Dynamics and Chaos – Group Work

Professor:

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Richardson Arms Race Model

We are going to analyze the Richardson Arms Race Model, as given in the paper on the back of this sheet.

- First find the fixed points for this model. This time they are not at $(0, 0)$.
- Write the system in Matrix form: $\vec{x} = A\vec{x} + b$ where b contains those extra constant terms
- Then find the trace and determinant of the matrix A . Discuss what kind of fixed points you would expect for different parameter values and what this means in terms of the arms race system.
- Using PPlane on the classroom computers investigate the system for a wide range of parameters. What are some good scenarios? What are some bad scenarios? What do the parameter values tell you about the countries involved AND the initial conditions.
- Choose TWO very different scenarios and prepare them for whiteboard presentation to the class. You should discuss what parameters you picked, what that means in terms of the countries, what the eigenvalues are, and finally show a phase portrait and discuss different trajectories.

Modifying the Richardson Arms Race Model With a Carrying Capacity

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Abstract: We endeavored to modify the Richardson Arms Race Model by introducing a carrying capacity term to each equation. These carrying capacity terms parallel the carrying capacity term introduced in a logistic growth model. As a result of these terms, new nullclines are created, thus drastically altering the resultant direction fields. We found that introducing these terms allowed us to predict the level of armament for each country upon the break of war.

I. Introduction

Lewis Fry Richardson, an English physicist, developed his arms race model after serving for France's medical corps in World War I. Deeply troubled by the events of World War I and the subsequent World War II, Richardson sought out a model that could predict large-scale military conflicts. Assuming that an arms race would be the platform from which a war was launched, he set out to model how one country's arms buildup affected the arms buildup of its opponent.

II. Richardson's Model

Richardson proposed the following system of differential equations to model an arms race between nation x and nation y :

$$dx/dt = ay - mx + r \quad (1a)$$

$$dy/dt = bx - ny + s \quad (2a)$$

In this model, x is the arms expenditure for nation x at time t and y is the arms expenditure for nation y at time t . The constants a and b represent the reactions of nations x and y to the arms level of the other nation. For example, for every unit of currency nation y spends on its arms supply, then nation x increases its arms spending by a . The constants m and n are the "fatigue" terms, representing the reluctance of nations x and y to spend more of their budget on arms. To use economic terms, m and n represent the desire of nations x and y to produce butter rather than guns. The constants r and s are the hostility/peace terms. A value of r less than 0 indicates that nation x has peaceful intentions toward nation y (i.e., x will decrease if the other terms in (1a) are 0). A value of r greater than 0 indicates that nation x has hostile intentions toward nation y (i.e., x will increase if the other terms in (1a) are 0).