Nonlinear Dynamics and Chaos - Group Work

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Lovable Lions vs Charming Cheetahs

Your task is to analyze the Lotka-Volterra model of competition between species. The Lovable Lions and Charming Cheetahs share a large territory, where they fend for themselves and eat off the land. They are at the top of the food chain and have no predators and lets assume that they do not eat each other, however they are competing for the same food supply, sometimes even stealing food from each other. The total amount of food is limited to what the land can support, zebras, buffalo, gazelles, etc. Each of the species has a carrying capacity, so in the absence of the other species they would grow to their carrying capacity. The Cheetahs have a higher intrinsic growth rate but the Lions are bigger and can more easily steal food from the Cheetah. The interaction between the species is proportional to the size of each of the species. This leaves us with the following equations:

$$\dot{x} = ax - x^2 - mxy$$
$$\dot{y} = by - y^2 - nxy$$

where $x(t)$ is the number of Lions and $y(t)$ is the number of Cheetahs. The growth rates for each species a and b tell how quickly the species reproduce. We usually think of these as positive numbers since a negative number would mean that even in the best circumstances the species would die off. The interaction terms for each species m and n tell how the interaction between species effects the population. Notice the negative in front, this implies that positive values of m or n mean a decline due to the interaction of the species. In general we will assume that our parameters are positive.

Our goal is to analyze this system using the tools from in class. Our first big question is: Is coexistence possible for our snuggly felines?

PART 1

- First find the nullclines for this model. Find the x and y intercepts for these equations.
- Draw the nullclines on a graph of y vs. x, assuming only positive values for the populations. Can you find a "coexistence" fixed point. (This is a fixed point where both populations can have a positive non-zero population) You should have four cases leading to two distinct cases for your fixed points.
- What are your fixed points for this model, solve for them in general. Remember, common denominators are your friend!

PART 2

Now assume that $a = 1$, $b = 3$, $m = 1$ and $n = 2$. What does this assume about the animals? In other words, who is is better at growing and who is better at competing? Now analyze the system for these parameters.

- Solve for the values of the fixed point and talk about what each one means.
- Find the Jacobian for the system.
- Analyze the stability of each of the fixed points.
- Sketch the phase plane for this system and discuss what happens for the different initial conditions. Is there any theoretical way to reach the "coexistence" fixed point?
- On your other white board, present your findings. Explain things in terms of the animal populations.

Extra - if time - Can you use the Jacobian and analyze the stability of the fixed points in the general case. In other words, look at the fixed points and stability without specifying number values for the parameters. Are there parameter values that change the dynamics of the system?