Nonlinear Dynamics and Chaos - Group Work

Professor:
Dr. Joanna Bieri
joanna_bieri@redlands.edu

NOTES - In your group define each of the following in your own unique words:

- Fixed Point
- Stable Fixed Point
- Unstable Fixed Point
- Autonomous First Order ODE
- Phase Portrait

DISCUSS Can you imagine a real world example that would have fixed points. Explain what the fixed points are in that system and whether they are stable or unstable. Make sure each member of the group can describe each of the definitions from today.

NOTES - For for following differential equation:

$$\dot{x} = f(x)$$

Write down mathematical descriptions for a fixed point and how you could determine whether or not the fixed point is stable or unstable. There are multiple ways to test stability of fixed points, can you come up with one algebraic and one graphical way?

DISCUSS As a group make up a nonlinear autonomous ODE. Make sure it has at least two fixed points and is somewhat interesting.
WHITEBOARD Write your group's equation on the white board and do a full analysis of the fixed points and their stability.

- Draw the flow on a line and identify the fixed points.
- Determine whether the fixed points are stable or unstable.
- Draw a phase diagram with multiple trajectories.
- Identify a few of the trajectories and describe in words what the system does in time. In other words, choose an initial condition (give approximate values) and then describe how the system changes as time evolves.
- Show how the phase diagram and the flow on the line relate.

CHECK IN Does each member of the group feel comfortable with the material? Do you have any bigger questions for Joanna or the class? Are you noticing any shortcuts or do you have any hints for your classmates solving these types of problems?