

# Nonlinear Dynamics and Chaos – Group Work

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Professor:

Dr. Joanna Bieri

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## Linear Stability Analysis - Group 1

### WHITEBOARD

Divide your whiteboard space into two sides or areas. We will completely analyze the problem below twice using the methods we have so far:

- 1. Geometric or Graphical approach - this is what we did yesterday! Graph the flow on the line, find the fixed points, and draw the Phase Portrait.
- 2. Linear Stability approach - this is what we talked about today! Algebraically find the fixed point and use calculus to determine the stability.

Consider the equation:

$$\dot{x} = a + x^2$$

where  $a$  is a constant that can be positive, negative, or zero. Discuss the fixed points and stability of this system for all three cases. What happens to the system for different values of  $a$ ?

*Whiteboard hints* When putting your presentation up on the whiteboard try to section things off so it is easy to follow your work. For example, draw your graph and any words you need to help describe it, then draw a rectangle around that to preserve some white space around it. Imagine that you are drawing a power point presentation on the whiteboard one slide at a time. Also remember you are welcome to take photos of the group work solutions to help you with your homework.

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## Linear Stability Analysis - Group 2

### WHITEBOARD

Divide your whiteboard space into two sides or areas. We will completely analyze the problem below twice using the methods we have so far:

- 1. Geometric or Graphical approach - this is what we did yesterday! Graph the flow on the line, find the fixed points, and draw the Phase Portrait.
- 2. Linear Stability approach - this is what we talked about today! Algebraically find the fixed point and use calculus to determine the stability.

Consider the equation:

$$\dot{x} = a - x^2$$

where  $a$  is a constant that can be positive, negative, or zero. Discuss the fixed points and stability of this system for all three cases. What happens to the system for different values of  $a$ ?

*Whiteboard hints* When putting your presentation up on the whiteboard try to section things off so it is easy to follow your work. For example, draw your graph and any words you need to help describe it, then draw a rectangle around that to preserve some white space around it. Imagine that you are drawing a power point presentation on the whiteboard one slide at a time. Also remember you are welcome to take photos of the group work solutions to help you with your homework.

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## Linear Stability Analysis - Group 3

### WHITEBOARD

Divide your whiteboard space into two sides or areas. We will completely analyze the problem below twice using the methods we have so far:

- 1. Geometric or Graphical approach - this is what we did yesterday! Graph the flow on the line, find the fixed points, and draw the Phase Portrait.
- 2. Linear Stability approach - this is what we talked about today! Algebraically find the fixed point and use calculus to determine the stability.

Consider the equation:

$$\dot{x} = ax - x^2$$

where  $a$  is a constant that can be positive, negative, or zero. Discuss the fixed points and stability of this system for all three cases. What happens to the system for different values of  $a$ ?

*Whiteboard hints* When putting your presentation up on the whiteboard try to section things off so it is easy to follow your work. For example, draw your graph and any words you need to help describe it, then draw a rectangle around that to preserve some white space around it. Imagine that you are drawing a power point presentation on the whiteboard one slide at a time. Also remember you are welcome to take photos of the group work solutions to help you with your homework.

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## Linear Stability Analysis - Group 4

### WHITEBOARD

Divide your whiteboard space into two sides or areas. We will completely analyze the problem below twice using the methods we have so far:

- 1. Geometric or Graphical approach - this is what we did yesterday! Graph the flow on the line, find the fixed points, and draw the Phase Portrait.
- 2. Linear Stability approach - this is what we talked about today! Algebraically find the fixed point and use calculus to determine the stability.

Consider the equation:

$$\dot{x} = ax + x^3$$

where  $a$  is a constant that can be positive, negative, or zero. Discuss the fixed points and stability of this system for all three cases. What happens to the system for different values of  $a$ ?

*Whiteboard hints* When putting your presentation up on the whiteboard try to section things off so it is easy to follow your work. For example, draw your graph and any words you need to help describe it, then draw a rectangle around that to preserve some white space around it. Imagine that you are drawing a power point presentation on the whiteboard one slide at a time. Also remember you are welcome to take photos of the group work solutions to help you with your homework.

## Thought Experiment - All Groups

### DISCUSSION

(The leaky bucket) The following example (from Hubbard and West 1991, p.159) shows that in some physical situations it is ok to have a non-unique solution. Non-uniqueness is natural and obvious, not pathological.

Consider a water bucket hanging from a tree with a hole in the bottom. You are walking down the street and you see this hanging bucket empty with a puddle beneath it, can you figure out when the bucket was full? Why or Why not?

Let's consider a differential equation that describes water leaking from a bucket:

$$\dot{h} = -C\sqrt{h}$$

Here

$$C = \sqrt{2g} \frac{a}{A}$$

where  $h$  represents the height of water in the bucket,  $g$  is the gravitational constant,  $a$  is the area of the hole in the bottom of the bucket, and  $A$  is the cross-sectional area of the bucket.

Solve this equation using separation of variables for the case of  $h(0) = 0$ , at  $t = 0$ , this is the initial condition when we found the bucket it was empty. What are the possibilities? What do they mean physically, as we go backwards in time? Try to explain how these solutions make sense.