Nonlinear Dynamics and Chaos - Group Work

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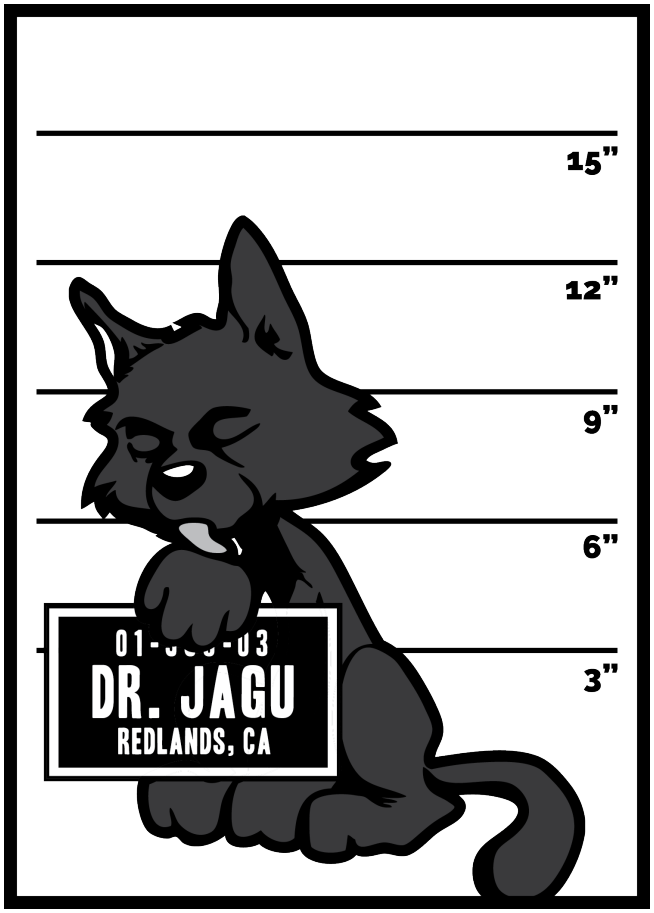
Fun with Lasers! - You'll shoot your eye out!

The universe's greatest evil mastermind, Dr. Jagu, has stolen one of earth's most powerful solid state lasers. This laser can emit a single shot energy of around 2,000,000 Joules delivered in under 1 nanosecond for a peak power of over 2,000,000,000,000,000 watts.



Cat image courtesy http://www.colouringbook.org

These lasers work by embedding "laser active" atoms in a solid state matrix and bounding them by partially reflecting mirrors. An external energy pump is then used to excite the atoms. When the energy pump is weak, the atoms are not excited out of ground state and the laser acts like an ordinary lamp, emitting randomly phased light. By increasing the strength of the energy pump above some laser threshold the atoms oscillate in phase and the lamp becomes a laser. This coherence develops because of cooperative interaction among the excited atoms.

The simplified, non-quantum, mathematical model is given here:

$$\dot{n} = gain - loss = GnN(t) - kn$$

where $n(t)$ is the number of photons in the laser field, N is the number of excited atoms, $G > 0$ is the laser gain coefficient, and $k > 0$ is a rate constant that has to do with the lifetime of a photon. Note that G and k are hard set by the type of laser used. The number of excited atoms will be reduced by the laser action, because the atoms emit photons, so we can relate the number of excited atoms to the emission of photons by

$$N(t) = N_0 - \alpha n$$

where N_0 is the number of excited atoms due to energy pumping and $\alpha > 0$ is the rate at which atoms go back to the ground state. Dr. Jagu is trying to get energy to run the energy pump in her laser by secretly tapping into the power grid at a variety of locations. We would like to make a targeted attack

of the power lines going to her evil lair so that she does not have enough power to get the laser up and running. However, we don't want her to immediately know that we cut the power, so we can catch her in action and put her away for life!

Your job is to make a plan of attack to stop this evil cat mastermind! Is there a way to allow her some power without activating the laser? How much power does she need to destroy life as we know it?

As a group you should:

- Write down the first order equation for $n(t)$ with N subbed in, and make sure you can define and understand all the variables.
- Find the fixed points and try to explain how the parameter N_0 affects the fixed points.
- Graph some examples of flow on the line and explain what the different cases mean and when the qualitative behavior changes.
- Draw the bifurcation diagram and discuss what solutions do in the different regions of the plot.

Each group will present one of these parts.