

Nonlinear Dynamics and Chaos – Group Work

Professor:

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Alien Outbreak

A certain alien species has landed on the planet and is planning to take over the world. This alien, the Brain Slug, is a parasite that sucks on the brains of its victims and can kill an entire city of in about four years. The brain slug has only one known earth predator, the mighty Eagle, a bird of prey that eats a variety of species but loves to dine on Brain Slug. As mathematical heroes we are going to model this system and see if there is any hope for the human race.

We begin by modeling the Brain Slug population logistically with some additional predation:

$$\dot{N} = RN\left(1 - \frac{N}{K}\right) - P(N)$$

here R is the growth rate, K is the carrying capacity, and $N(t)$ is the Brain Slug population. Our predation term is given by

$$P(N) = \frac{BN^2}{A^2 + N^2}$$

where $A, B > 0$. This means that if the Brain Slug population is too small the Eagle will seek food elsewhere, however as the Brain Slug population increases the Eagle returns and eats all it can. This leaves us with the following equation:

$$\dot{N} = RN\left(1 - \frac{N}{K}\right) - \frac{BN^2}{A^2 + N^2}$$

Here we see **four** parameters R, K, B , and A . We are going to non-dimensionalize the system to combine some of these parameters. (IN CLASS) Leaving us with the following system to analyze:

$$\dot{x} = rx\left(1 - \frac{x}{k}\right) - \frac{x^2}{1 + x^2}$$

Your job is to analyze this system! Let's start together in class with non-dimensionalization.

Ok, now that we have proved that this is the non-dimensional form, please fully analyze the system.

$$\dot{x} = rx\left(1 - \frac{x}{k}\right) - \frac{x^2}{1 + x^2}$$

where $r = \frac{AR}{B}$ and $k = \frac{K}{A}$.

Here are some hints for the steps to take...

- Solve for the fixed points - You should be able to show that one fixed point, $x^* = 0$ always exists and the other is defined by the equation

$$r - \frac{r}{k}x^* = \frac{x^*}{(1 + x^{*2})}$$

- For the equation - Plot the curves and talk about the number of fixed points for a variety of parameters. Good parameter ranges to start with are $0 < r < 1$ and $0 < k < 10$. Can you see when a bifurcation might occur?
- Describe in words what you see. For example, when k is small we see we have what number of fixed points?
- Look at the Flow on a line for the same range of parameters and see if you can describe the stability of the fixed points.
- Talk about the different cases and explain what is happening to the brain slug outbreak.
- If $r = .5$ and $k = 100$ for our population - what does your group suggest we do?
- (EXTRA IF TIME) These fixed points occur when the two lines on the graph are both equal and tangent to each other. Use this information to make two equations for two unknowns to solve for r as a function of x and k as a function of x . Plot these functions as parametric curves (on foo plot) for a range of x values. In other words, plot the points $(r(x), k(x))$ on an r vs. k graph.

This group lab is taken from our textbook - see the Spruce Budworm insect outbreak problems for more information.