Nonlinear Dynamics and Chaos - Group Work

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Analysis of an Uncoupled Linear System

Together we just saw a linear system that had closed circular orbits in the phase plane. Now we will investigate a different system to see what other possible curves we might see in the phase plane.

Consider the system:

$$\dot{x_1} = ax_1, \qquad \dot{x_2} = -x_2$$

where a is a parameter that can take on any real values. What is the fixed point?

Your job is to plot the phase plane for a variety of a values as see if you can come up with ALL of the qualitatively different solutions possible for this system and ranges for a. For each new type of phase plane do the following:

- Draw the phase plane in your notes or on this paper. (copy what you see in the Matlab graph) Make sure that you include arrows to show the direction of the solution.
- Say in words what is happening to your solution. EX. When I start at $(1, 0)$ my solution goes directly to the fixed point along the x_2 axis with no change in x_1, but if I start at $(1, 1)$ my solution decays to the fixed point decaying faster in the x_1 direction than in the x_2 direction.
- Solve the system of equations (you can solve each separable equation independently).
- Do your pictures make sense base on the solution you just found? (Hint: talk about the parameter a and compare the exponents.)

When we get back together as a class we will develop vocabulary to identify each of these different looking graphs. So leave room on your paper to add descriptions during lecture.