Day 1 Nonlinear Dynamics 2014.

Dynamics is the study of systems that change in time.
This covers a wide range of topics!

Classical or Quantum Mechanics
Thermodynamics
Fluid mechanics
Chemical Reactions etc.

- OLD GOAL → to find the answer, exact position or closed form solution for all time.

Not always possible or even practical!

- NEW GOAL → get more qualitative predictions about what will hapen in the system.

seek long term solutions — will the system eventually settle down to a single soln?

look for instabilities — will the system spontaneously oscillate.

investigate chaos — does the system exhibit complicated dynamics with sensitivity to initial conditions.

Even seemingly simple systems exhibit complicated behavior!!

Ex The logistic equation:

$$x_{n+1} = r x_n (1 - x_n)$$

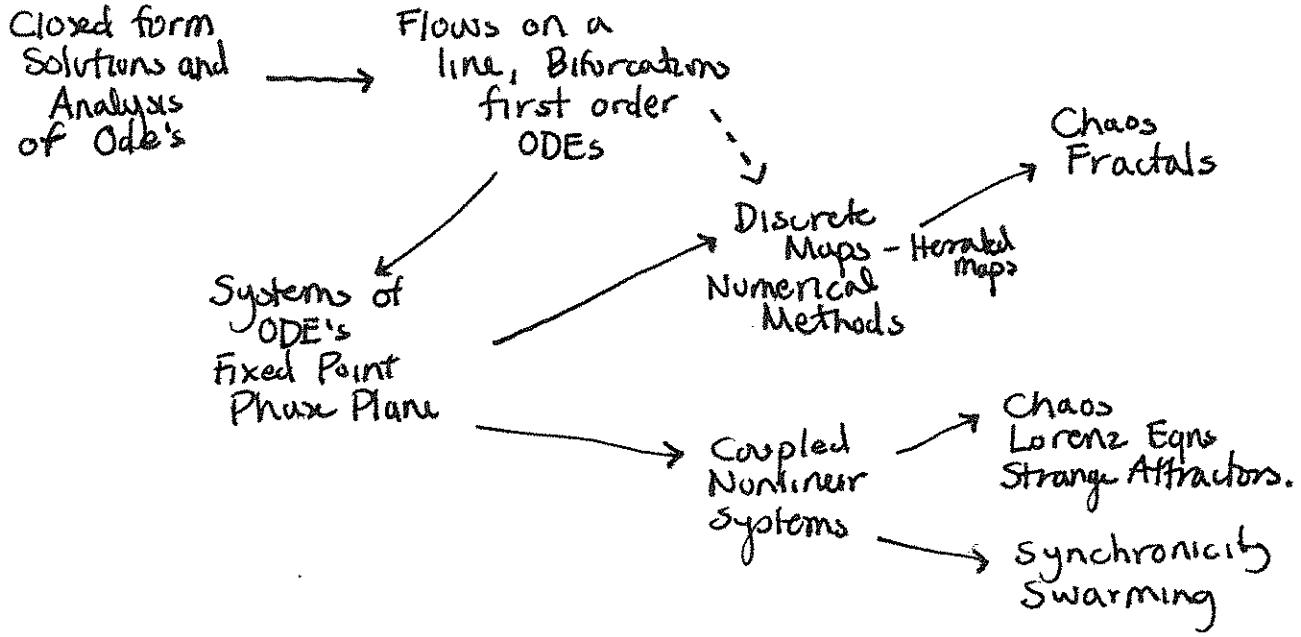
— this seem like a pretty simple, discrete, nonlinear equation.

lets see what happens for different values of r.

Small differences in the initial conditions lead to widely diverging solutions — even in the absense of stochastic events!

GOAL – start building the mathematical background to understand the mathematics behind these amazing phenomenon.

History and Context.



Important Terms and Definitions:

Differential Equation (continuous)

$$\frac{dx}{dt} = f(x,t)$$

$$\dot{x} = f(x,t)$$

Difference Equation (discrete) Iterated Map

$$x(t+1) - x(t) = f(x,t)$$

Discrete Numerical Methods

We will talk mostly about Differential Equations.

Types of Differential Equations:

Ex $m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$ || $m\ddot{x} + b\dot{x} + kx = 0$ alternate notation.

Order ⇒ number of initial conditions needed number of the highest derivative

second order – $\ddot{x}$

Linear vs Nonlinear ⇒ Linear equations – all dependent variables appear as linear combinations!

linear – dont see $\dot{x}^2$ or $\sin(x)$ or $x\dot{x}$

ordinary differential equation ⇒ ODE's have only one independent variable.

Ex $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$

Second Order, Linear, Partial Differential Equation.

Ex $\dot{x} = rx(1-x)$

First Order, nonlinear, Ordinard Differential Equation.

GROUPS – Come up with 3-4 challenging and different examples of differential equations.

Classify them... but keep it secret.

Pass your equations – No Solns to another group – see if they can correctly classify the eqns.

IN THIS CLASS → We focus on FIRST ORDER SYSTEMS !!

System ⇒ group of first order equations solved together.

Why are Nonlinear Problems so hard?

When dealing with linear things we can solve each piece separately and then put the parts back together

Ex Integration!

$$\int x^2 + \sin(x) + e^x + \sqrt{x}\, dx$$

addition is a linear operator so we can consider the pieces seperately.

VS

$$\int x^2 \sin(x) e^x \sqrt{x}\, dx$$

multiplication is nonlinear — now we deal with this all together!

BREAK!

We are only considering first order equations — Does this limit us?

NO!

Any higher order differential equation can be reduced to and equivalent system of first order equations!

EX Consider the second order system:

$$\frac{d^2x}{dt^2} = \ddot{x} = f(x, t)$$

define new variables: x_1 and x_2 and let

$$\dot{x}_1 = x_2 \quad \text{then} \quad \dot{x}_2 = \ddot{x}_1$$

just take the derivative.

then say $\ddot{x}_1 = f(x_1, t)$ this is just the ode we started with.

BUT this means that

$$\dot{x}_2 = f(x_1, t)$$

so our equivalent system is:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = f(x_1, t)$$

We could generalize this to n^{th} order equations

#1 Write in correct form. $$\frac{d^n x}{dt^n} = f\left(\frac{d^{n-1}x}{dt^{n-1}}, \dots, \frac{dx}{dt}, x, t\right)$$

#2 define new variables: $x_1, x_2, x_3 \dots x_n$

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = x_3, \quad \dot{x}_3 = x_4, \quad \dots \quad \dot{x}_{n-1} = x_n$$

but then $\dot{x}_n = \ddot{x}_{n-1} = \dddot{x}_{n-2} = \dots \frac{d^n x_1}{dt^n}$

#3 So $\dot{x}_n = f(x_{n-1}, x_{n-2}, \dots, x_2, x_1, t)$

#4 The equivalent system:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3$$
$$\vdots$$
$$\dot{x}_n = f(x_{n-1}, x_{n-2}, \dots x_2, x_1, t)$$

Group 1 -

For each of the follow differential equations:

- What is the independent variable?
- What is the dependent variable?
- List all parameters.
- Classify the differential equation.
- Reduce it to the equivalent first order system.

$$\frac{d^2x}{dt^2} + mx = 0$$

$$\frac{d^4y}{ds^4} - \sin\left(\frac{dy}{ds}\right) = m$$

For each problem that you do neatly write up a solution on the whiteboard that includes lots of words explaining what you are doing in each step and why it makes sense. We will be presenting these solutions to the class and they will be posted online to help with homework. Each member of the group (not the group leader) must participate in the presentation.

Group 2 -

For each of the follow differential equations:

- What is the independent variable?
- What is the dependent variable?
- List all parameters.
- Classify the differential equation.
- Reduce it to the equivalent first order system.

$$\frac{d^2x}{dt^2} + \frac{g}{L}\sin(x) = 0$$

$$\frac{d^3z}{dt^3} + bz = e^t$$

For each problem that you do neatly write up a solution on the whiteboard that includes lots of words explaining what you are doing in each step and why it makes sense. We will be presenting these solutions to the class and they will be posted online to help with homework. Each member of the group (not the group leader) must participate in the presentation.

Group 3 -

For each of the follow differential equations:

- What is the independent variable?
- What is the dependent variable?
- List all parameters.
- Classify the differential equation.
- Reduce it to the equivalent first order system.

$$\frac{d^2x}{dt^2} - x^2 = e^t$$

$$\frac{d^3p}{dx^3} + \frac{d^2p}{dx^2} - \frac{b}{a}x = 0$$

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Group 4 -

For each of the follow differential equations:

- What is the independent variable?
- What is the dependent variable?
- List all parameters.
- Classify the differential equation.
- Reduce it to the equivalent first order system.

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + kx = F\cos(t)$$

$$\frac{d^5y}{ds^5} - \frac{dy}{ds} = 0$$

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