Nonlinear Dynamics and Chaos - Week 3 Homework

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DAY 1

For the following linear systems: First write the systems in matrix form. Then use Pplane to draw the phase portrait. For each problem classify the type of fixed point using the vocabulary that we developed in class. Discuss what happens to your solution if you start at $(x_0, y_0) = (1, 0)$

1. $\dot{x} = y, \quad \dot{y} = -2x - 3y$
2. $\dot{x} = 5x + 10y, \quad \dot{y} = -x - y$
3. $\dot{x} = 3x - 4y, \quad \dot{y} = x - y$
4. $\dot{x} = -3x + 2y, \quad \dot{y} = x - 2y$
5. $\dot{x} = -y, \quad \dot{y} = -x$

DAY 2

Find the eigenvalues and eigenvectors for the linear systems. Classify the fixed point using the eigenvalues. Compare your results to what you found on Day 1 (these are the same four systems that you should already have written in matrix form).

1. $\dot{x} = y, \quad \dot{y} = -2x - 3y$
2. $\dot{x} = 5x + 10y, \quad \dot{y} = -x - y$
3. $\dot{x} = 3x - 4y, \quad \dot{y} = x - y$
4. $\dot{x} = -3x + 2y, \quad \dot{y} = x - 2y$
5. $\dot{x} = -y, \quad \dot{y} = -x$

For the following linear systems: First write the systems in matrix form. Then Classify the fixed point based on the trace and determinant of the matrix. Finally, find the eigenvalues and eigenvectors for the system and draw a reasonably accurate phase portrait. Discuss what happens to your solution if you start at $(x_0, y_0) = (1, 0)$

6. $\dot{x} = 5x + 2y, \quad \dot{y} = -17x - 5y$
7. $\dot{x} = -3x + 4y, \quad \dot{y} = -2x + 3y$
8. $\dot{x} = 4x - 3y, \quad \dot{y} = 8x - 6y$
9. $\dot{x} = y, \quad \dot{y} = -x - 2y$

DAY 3

Use a computer (aka PPLANE) to plot the phase portrait of the following nonlinear systems. Write in words what you seen in the phase portrait. What different types of solutions might you expect. In each case graphically find the fixed points and discuss their stability based on the phase portrait.

1. $\dot{x} = y, \quad \dot{y} = -x + y(1 - x^2)$
(van der Pol oscillator)

2. $\dot{x} = 2xy, \quad \dot{y} = y^2 - x^2$
(Dipole fixed point)

3. $\dot{x} = y + y^2, \quad \dot{y} = -\frac{1}{2}x + \frac{1}{5}y - xy + \frac{6}{5}y^2$
(Two-eyed monster)

4. $\dot{x} = y + y^2, \quad \dot{y} = -x + \frac{1}{5}y - xy + \frac{6}{5}y^2$
(Parrot)

For each of the following systems: Find all possible fixed points, linearize the system near these fixed points and classify them, sketch the phase portrait near the fixed points, and finally try to fill in the phase portrait as best you can. Remember to check your answers using Pplane but please attempt to sketch the solution BEFORE checking with Pplane.

5. $\dot{x} = x - y, \quad \dot{y} = x^2 - 4$
6. $\dot{x} = \sin y, \quad \dot{y} = x - x^3$
7. $\dot{x} = y + x - x^3, \quad \dot{y} = -y$
8. $\dot{x} = 1 + y - e^{-x}, \quad \dot{y} = x^3 - y$

Short Answer

9. Explain in words what each of the following things are, how you find (or draw) them and what they tell you about a nonlinear system. Give an example of each.

- One Dimensional Flow on a Line
- Fixed Point
- Potential Function for 1-D system
- Bifurcation Diagram
- Phase Portrait (2-D)
- Saddle-Node Bifurcation
- Transcritical Bifurcation
- Pitchfork Bifurcation (sub and supercritical)
- Stable and Unstable Manifold
- Euler Method, Improved Euler Method, and Runge Kutta

DAY 4

Choose your own adventure!
CHOOSE ONE of the sections of the book to read and understand.

1. Rabbits vs Sheep - Chapter 6.4 - This is an example of the predator prey model and an analysis of the fixed points of the system. This is a great example for people with a general interest in science or for those who want a fairly straightforward example of fixed points and stability in a real system.
   - Read the chapter and takes nodes - doing the example.
   - Explain, what is a basin of attraction and what are separatrices
   - Then do problems 6.4.1 and 6.4.2 from the book.
2. Conservative Systems - Chapter 6.5 - This is a great chapter for people interested in physics and engineering. Much of classical mechanics is based on the ideas of conservation. This is a slightly more challenging concept.
   - Read the chapter and takes nodes - doing the examples.
   - What does it mean for a system to be conservative and what is a homoclinic orbit?
   - Then do problems 6.5.1 and 6.5.2 from the book.
3. Index Theory - Chapter 6.8 - This is a great chapter for people interested more of the mathematical theory behind some of our concepts. Index theory gives is global information about the phase plane. This basic idea is straightforward, but the mathematical generalizations are a bit more difficult.
   - Read the chapter and takes nodes - do the examples and explain why the proofs/theorems make sense.
   - Explain what the index of a closed curve is and what the index can tell you about the phase plane.
   - Then do problems 6.8.2 and 6.8.7 from the book.