

Numerical Analysis - Homework 12

Professor:

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Office Hours:

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Homework Problems

- Problems 1 and 4 in chapter 4.2
- Non book problems below

You should attempt all these problems before class, but we will work on them in class with our groups.

Non Book Problems

Approximate the following functions using an interpolating polynomial $P_n(x)$.

- (a) $f(x) = \sqrt{1+x^2}$
- (b) $f(x) = \frac{1}{1+25x^2}$

1. Write computer code to approximate the functions on the range $[-1, 1]$ using a set of $n+1$ evenly spaced nodes. Take $n = 5, 10, 25$.
2. Plot $P_n(x)$ and $f(x)$ on the range $[-1, 1]$. How does the approximation compare with the actual function.
3. Consider the error $E(x) = f(x) - P_n(x)$ plot the error for each approximation. Does the accuracy of the approximation get better or worse as you increase n ?
4. Repeat problems 1-3 for $n+1$ nodes that are not evenly spaced. Use:

$$x_k = \cos\left(\frac{\pi(k-1)}{n}\right), \quad k = 1, 2, \dots, n+1$$

to generate the $n+1$ nodes.

5. Using the error bounding formula from in class. Consider

- (a) $f(x) = \sqrt{1+x^2}$
- (b) $f(x) = \frac{1}{1+25x^2}$

Bound the error in the linear interpolation $P_1(x)$ of these functions on $[-1, 1]$. Part 5 is handwritten work - not programmed in Jupyter.