

Numerical Analysis - Homework 17

Professor:

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Office Hours:

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Homework Problems

For this homework you will write your own code for numerical differentiation. Please reach out if you are struggling with this and I can provide some pointers and some sample code.

- Use Taylor's Polynomials and Undetermined Coefficients to prove the following approximate differentiation formulas:

$$f'''(x) \approx \frac{1}{2h^3} [f(x+2h) - 2f(x+h) + 2f(x-h) - f(x-2h)]$$

$$f'(x) \approx \frac{1}{6h} [2f(x+3h) - 9f(x+2h) + 18f(x+h) - 11f(x)]$$

- For each of the approximations in number 1, what is the error? What order approximations are these? What do you expect the ratio of convergence of solutions to be?
- What does the following difference scheme approximate and what is the error?

$$\frac{1}{2h} [f(x+3h) + f(x-h) - 2f(x)]$$

- Write code to calculate forward, backward and centered difference. Use your code to find the derivative of $f(x) = e^x$ at $x = 0$. Discuss how the error decreases each time you cut h in half. Compare this to the order of the method. How does this compare to the expected error? For example, in forward difference method we developed the error formula 5.77 in the book. If you plug in $c=0$ to that formula for different h values what would you expect your error to be?
- Write code that calculates the second derivative using the method we derived in the class video, this is 5.91 in your book. This code should be similar to the first derivative code. Use your code to find the second derivative of $f(x) = e^x$ at $x = 0$ for $h = 0.1, 0.05, 0.025, 0.0125$. Discuss how the error decreases each time you cut h in half. Compare this to the order of the method. How does this compare to the expected error?