Numerical Analysis - Homework19

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Homework Problems
Write code for Euler's Method and Error Approximation using Richardson's Extrapolation

- Do problems 1 (a,b,c) and 2(all) from chapter 8.2.
- Non Book problem: Use Euler's Method to solve:

$$y' = -ye^{-x^2}, \quad y(0) = 1$$

  on the range $[0, 10]$. Approximate the error using Richardson's Method (extrapolation); because you don't know the true solution, approximate error is the best you can do. Have your code calculate the approximate error at some representative x-nodes, and then find the maximum. For what h-value is the maximum error less than 0.05. Change your initial condition to $y(0) = 0$. What happens to your solution and your error for this case? Explain what is happening here.

Ideas for exploration:

- For Problem 1 create a graph where you plot the real solution (given in the book as $Y(x)$ and your solution for $h = 0.05$. These can be plotted all on the same graph. Also, create a table of values for the error at a few x-locations. Since you are given the real values $ERROR = Real - Euler$. This should be similar to tables 8.1 and 8.2 in the text. Comment on the error and the ratio of errors as h decreases by half. Then calculate the error estimate based on Richardson's Extrapolation. How does this compare to the real error?
- For problem 2, start by discussing the conditioning of the ODE. What is the requirement for this ODE to be well-conditioned? You can plot your results in a series of graphs one graph for the solution in each of the cases. For example, one graph for all the h-values and $\lambda = -1$, one graph for all the h-values and $\lambda = 1$, etc. Discuss what you see in the graphs and whether or not the results make sense.
- For the NON-BOOK problem discuss what h-value you needed to get an error less than 0.05. What happened when you changed your initial condition. Do an analysis of the conditioning of this problem to give yourself some insight into what is happening here. Please discuss what you learned.

You should attempt all these problems before class, but we will work on them in class with our groups.