Numerical Analysis - Fixed Point Iteration

Consider the following iterations:

$$x_{n+1} = 5 + x_n - x_n^2 \tag{1}$$

$$x_{n+1} = \frac{5}{x_n} \tag{2}$$

$$x_{n+1} = 1 + x_n - \frac{1}{5}x_n^2 \tag{3}$$

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{5}{x_n}\right) \tag{4}$$

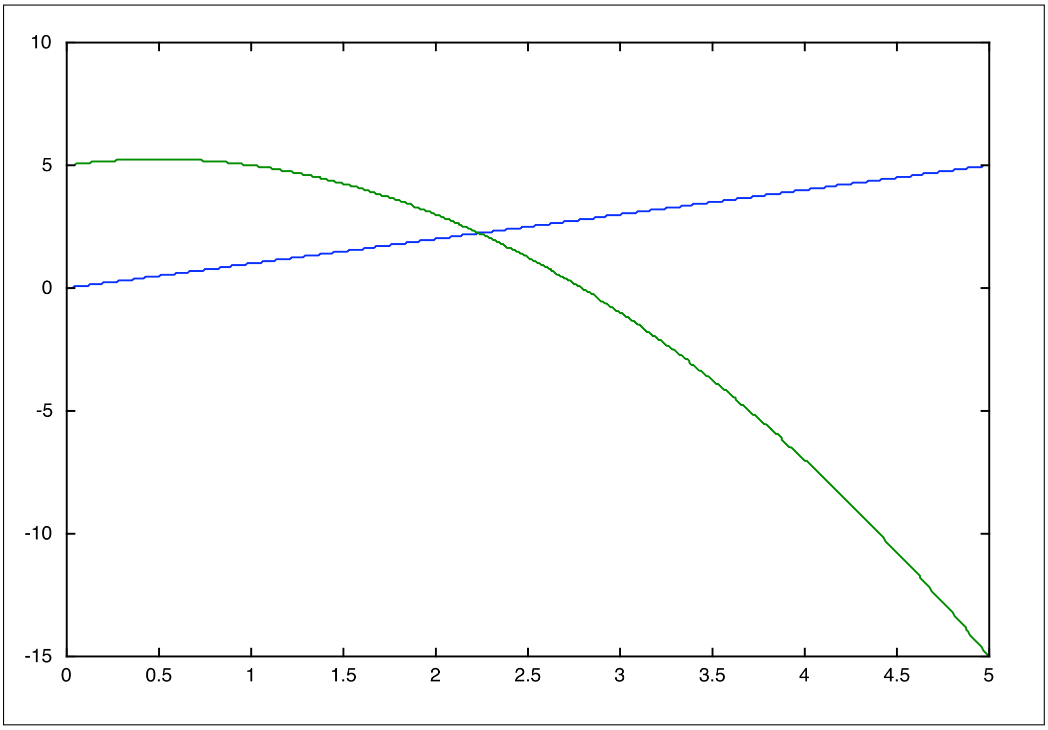
Show that $\alpha = \sqrt{5}$ is a fixed point for each of these iterations. Note that really the possibilities are endless. You could make up lots of iterations that would have $\alpha = \sqrt{5}$ as a fixed point.

Now using Python, write a short program that starts with $x_0 = 2.5$ and does $N = 5$ steps for the above iterations. Print out each of the steps and describe in words what is happening. NOTE - You can use this code in your homework!

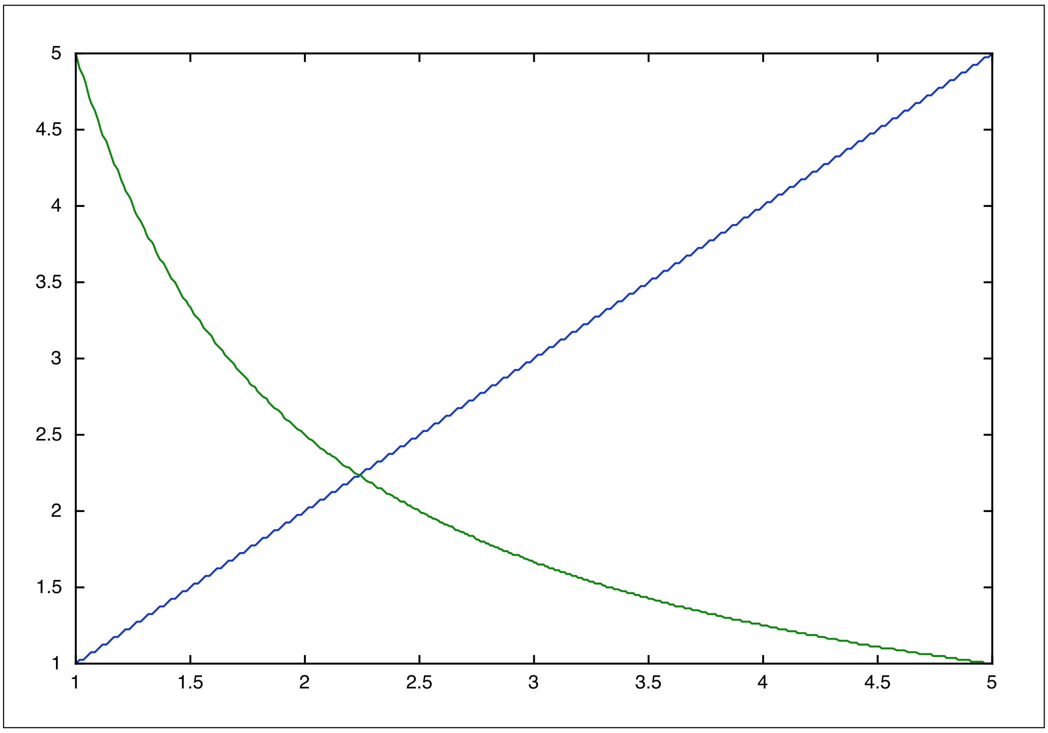
COBWEB DIAGRAMS
A graphical way to see the convergence or divergence of an iterative sequence OR a way to consider the stability of fixed points.

1. First choose a starting value x_0
2. Draw a vertical line from x_0 up to your $g(x)$
3. Draw a horizontal line to $y = x$, this is your x_1
4. Draw a vertical line from x_1 to your $g(x)$
5. Continue until you see what is happening.

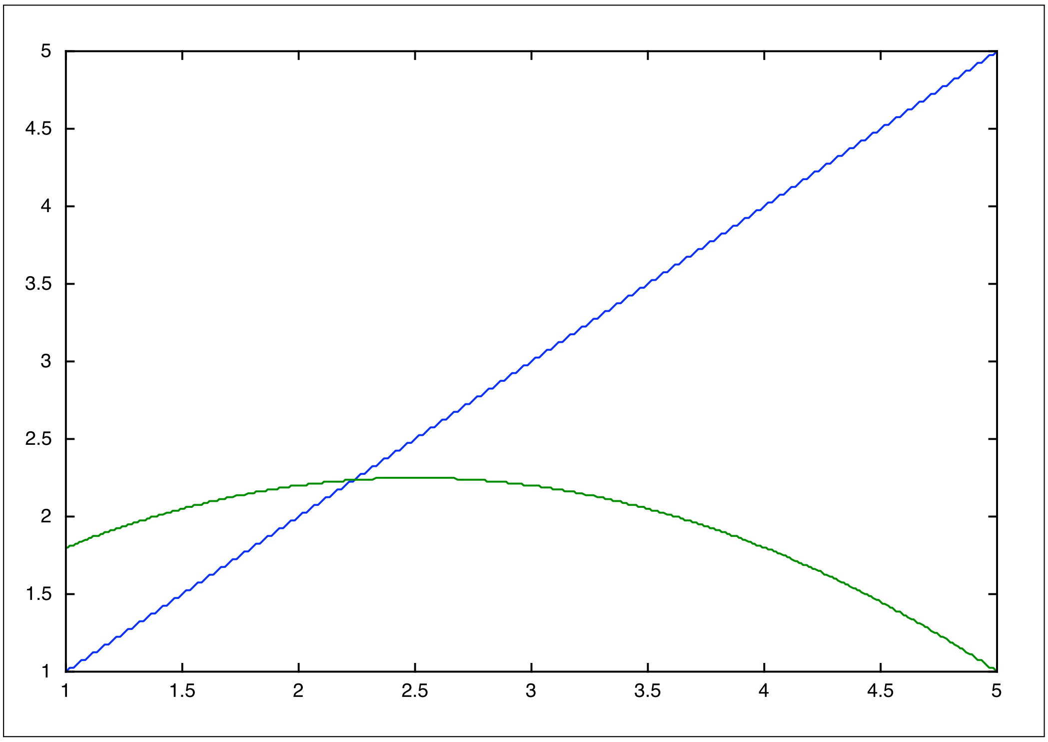
$x_{n+1} = 5 + x_n - x_n^2$



$x_{n+1} = \frac{5}{x_n}$



$x_{n+1} = 1 + x_n - \frac{1}{5}x_n^2$



$x_{n+1} = \frac{1}{2}\left(x_n + \frac{5}{x_n}\right)$