# Numerical Analysis - Day 1.

- Pass out syllabus - go over course.
- Introductions:
  - Name
  - One thing about yourself that we don't know.

  PHOTOS: Bring 3x5 cards.

1. Why study Numerical Analysis ... it is Awesome!

   There are more problems out there that cannot be solved by hand. Most applied problems require approximation and computer analysis.

2. What is Numerical Analysis:

   Finding approximate solutions to very difficult or unsolvable problems in mathematics.

   Computers help automate the numerical algorythms so we get more accurate approximations.

IN THIS CLASS.

- We will be doing a lot of programming. In class we will use FreeMat or Matlab.

  You can install FreeMat on your own personal computer.

  It is available on Math Department Computers.

- Numerical Methods are only a good as the Mathematician or Programmer.
  ALWAYS CHECK YOUR RESULTS.
  Are they logical? Do they make sense?

- Almost EVERY program that you write will have bugs or errors.
  You must be patient and persistent.

- Computers do not read minds.... your syntax and code must be exact.

- START EARLY writing and debugging code always takes longer than you think it will
  THEN after the code is working you still have to use it to complete the assignment.

- I plan to also teach you how to use <u>LaTex</u> to type up your project reports.
  - This will help in Capstone
  - This will help in Gradschool
  - A good thing to know.

Ch. 1.1 Taylor Polynomials (Taylor Series)

Numerical Analysis is all about replacing a difficult problem with an easier approximation.

Taylor Polynomials are hugely important in doing this.

The general Formula.

$$P_n(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!}f''(a) + \cdots + \frac{(x-a)^n}{n!}f^{(n)}(a)$$

$$= \sum_{j=0}^{n} \frac{(x-a)^n}{n!} f^{(n)}(a)$$

Then we say we can approximate $f(x)$ for $x$ near $a$ using $P_n(x)$.

Where does this come from ...

* Consider the function $f(x) = e^x$ and imagine we want to find $f(.01)$ BY HAND!

well we say ... I know $f(0) = 1$ and it must be close to this!

So we say...
Lets approximate $f(x) = e^x$ with a straight line.

$$p_1(x) = y = mx + b$$

<u>IN GENERAL</u> say we want this straight line to match $f(x)$ near $x=a$.

$$p_1(a) = f(a) \quad \underline{\text{and}} \quad p_1'(a) = f'(a)$$

1. plug in the conditions ...

$$P_1(a) = \boxed{ma + b = f(a)}$$
$$P_1'(a) = \boxed{m = f'(a)}$$

two eqns for two unknowns.

→ plugging in m.

$$m = f'(a) \quad \text{and} \quad b = f(a) - f'(a)a$$

2. plug into the eqn for a line...

$$\text{Then} \quad P_1(x) = f'(a)x + f(a) - f'(a)a$$

3. gather like derivatives.

$$P_1(x) = f(a) + (x-a)f'(a)$$

$$\text{or} \quad f(x) \sim f(a) + (x-a)f'(a)$$

LINEAR APPROXIMATION
OR $P_1(x)$ Taylor Series!

<u>For our function</u> $f(x) = e^x$ near $x=0$

$$P_1(x) = f(a) + (x-a)f'(a)$$

$$f(0) = 1 \qquad f'(0) = 1 \qquad a = 0$$

$$f(x) \sim 1 + x \quad \text{for } x \text{ near zero.}$$

AND $f(.01) \cong 1.01$

BUT WAIT ... we could do better than this !!
$f(x) = e^x$ is curvy ... lets try a quadratic.

IN GENERAL. $P_2(x) = b_0 + b_1 x + b_2 x^2$

we want to match a point, slope and curvature...

$$P_2(a) = f(a) \qquad P_2'(a) = f'(a) \qquad P_2''(a) = f''(a)$$

1. Plug in

$$\boxed{\begin{aligned} P_2(a) &= b_0 + b_1 a + b_2 a^2 = f(a) \\ P_2'(a) &= b_1 + 2b_2 a = f'(a) \\ P_2''(a) &= 2b_2 = f''(a) \end{aligned}}$$

three eqns
three unknowns.

$$b_2 = \frac{f''(a)}{2}$$

then $b_1 = f'(a) - f''(a)a$

and $b_0 = f(a) - f'(a)a + f''(a)a^2 \mp \frac{f''(a)a^2}{2}$

2. plug into the quadratic

$$P_2(x) = f(a) - f'(a)a + f''(a)a^2 \mp \frac{f''(a)}{2}a^2$$
$$+ f'(a)x - f''(a)ax + \frac{f''(a)}{2}x^2$$

3. gather like derivatives

$$P_2(x) = f(a) + (x-a)f'(a) + \frac{1}{2}(x^2 \mp 2ax + a^2)f''(a)$$
$$= f(a) + (x-a)f'(a) + \frac{1}{2}(x-a)^2 f''(a)$$

QUADRATIC APPROXIMATION
OR $P_2(x)$ TAYLOR SERIES!

For our function $f(x) = e^x$ near $x=0$

$$P_2(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2}f''(a)$$

$f(0)=1 \quad f'(0)=1 \quad f''(0)=1 \quad a=0$

$$f(x) \sim 1 + x + \frac{x^2}{2}$$

and $f(.01) \cong 1.01 + \frac{(.01)^2}{2} = 1.01005$

ON CALCULATOR $e^{.01} = 1.01005016708.$

If we kept going like this we would see our pattern and derive the formula for Taylor Polynomials.

GENERAL IDEA... when considering a difficult problem we try to replace it with a nearby easier problem.

Replace the evaluation of $f(x) = e^x$ with the evaluation of a polynomial $P_n(x)$.

Next Time Error.