

Differential Equations – Advanced Problem Set 11

Professor:

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Directions: Do the following book problems

Section 6.1 Problems 25, 26

Section 6.2 Problems 33, 34

Application Problem:

Consider the system of equations given here:

$$x_1' = \left(1 - \frac{x_1}{10}\right)x_1 - x_1x_2$$

$$x_2' = -x_2 + x_1x_2$$

Here x_1 is a Host species and x_2 is a Parasite species. Let's imagine that x_1 represents fish in an aquarium, where the aquarium was built to only sustain 10 fish, and x_2 represents a parasite that feeds off the fish and weakens the fish population. Your job is to:

1. Find the fixed points of the system.
2. Graph the phase plane and discuss the type and stability of the fixed points.
3. MOST IMPORTANT - Explain what each of the fixed points means in terms of the fish and parasite. Then discuss what the stability of the fixed points means in terms of the fish and parasite. You might want to think about what would happen to the system for different initial values.

Other Notes:

1. Problem 33 Introduces you to the idea of a **Bifurcation** where a small change in a parameter ϵ completely changes the location, type, or stability of the fixed point. Please read the introduction to the problem on page 389.