

Differential Equations – MAKE UP – Advanced Problems

Professor:

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Course Review

PART 1 – FIRST ORDER EQUATIONS

We will start by considering the following ODE and use this ode for the rest of PART 1:

$$y' = \frac{y^3}{x-3} \quad y(0) = 1$$

Please fully classify this ODE.

- What does $f(x, y)$ tell you about existence or uniqueness?
- What does $\frac{\partial f}{\partial y}$ tell you about existence or uniqueness?
- Comment on existence or uniqueness problems for the given ODE.

Please solve the ODE by separation.

Please solve the ODE using the exact method.

This week we have been working on solving ODEs numerically. Solve this ODE using EULER'S METHOD. We will approximate the solution at $y(2)$ for $h = .4, .1, .025, .00625$. How much are we decreasing our step size for the different h values? Based on these h values how much do you expect your error to decrease for the different h values? You must submit a link to some working code for this assignment.

Fill in the table.

h	$y(2)$	Exact Error	Approximate Error	Ratio of Approx Error
.4			----	----
.1				----
.025				
.00625				

Now use Euler's method to approximate $y(4)$ for our ODE.

h	$y(4)$	Exact Error	Approximate Error	Ratio of Approx Error
.4			----	----
.1				----
.025				
.00625				

What is going wrong here?

PART 2 – HIGHER ORDER EQUATIONS

Next we will consider the ODE and use this ode for the rest of PART 2:

$$y'' - 9y = e^{3x}, \quad y(0) = 0 \quad y'(0) = 1$$

Please fully classify this ODE.

Please do the following:

- Find the homogeneous solution using the characteristic equation.
- Find the nonhomogeneous solution using the Method of Undetermined Coefficients.
- Find the nonhomogeneous solution using Variation of Parameters.

Please solve the ODE using a Laplace Transform.

Please rewrite the ODE as a first order system.

Use the Eigenvalue Method to find the Homogeneous Solution. You should get the same answer! What would you need as your guess for the nonhomogeneous solution if you were doing Method of Undetermined Coefficients.

PART 3 – NONLINEAR EQUATIONS

Consider the ODE

$$y'' + \frac{1}{2}y' = -\sin(y)$$

Please fully classify this ODE. Then convert it into a First Order Nonlinear System.

Please find the fixed points of this system and classify their stability. You can also plot a Pplane and check your work!

PART 4 – A FUN CHALLENGE (Extra Bragging Rights!)

Consider the ODE

$$y'' + \frac{1}{2}y' = -\sin(y) \quad y(0) = \frac{\pi}{2}, \quad y'(0) = 0$$

This is the same ODE we just turned into a first order system, but now you have initial conditions. Apply Euler's Method to each of the first order equations and use Python to find an approximate solution for $y(10)$. Consider different h values and the error in your approximation.

Plot $x_1(t)$ vs. $x_2(t)$ to see the curves that you see in the Pplane!