Differential Equations - Exploration

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Exploring the Phase Plane and Fixed Points

Directions: Please complete the following problems before following along with the next lecture video

PART 1. (see next page to check answers)

Consider the simple linear system

$$\frac{dx}{dt} = -x$$
$$\frac{dy}{dt} = -ky$$

Do the following

- Find the one and only fixed point for the system
- Solve the system. You will have the parameter k in your final solution.
- Solve for the curves in the phase plane. In other words, using your solution above find y as a function of x.
- Now consider the following cases and talk about whether or not solutions are growing or decaying in each case.
  - $k = 1$
  - $k > 1$
  - $0 < k < 1$
  - $k < 0$

  What should the phase plane look like? Check you work using pplane OR https://aeb019.hosted.uark.edu/pplane.html
- Are solutions going to end up at the fixed point in all cases? If not are there special cases where they DO end up at the fixed point?

PART 2. (see next page to check answers)

Consider the simple linear system

$$\frac{dx}{dt} = y$$
$$\frac{dy}{dt} = -w^2 x$$

Do the following

- Find the one and only fixed point for the system
- Solve the system. You will have the parameter w in your final solution.
- For this system we can argue that trajectories in the phase plane are elipses:

$$\frac{x^2}{C^2} + \frac{y^2}{C^2 w^2} = 1$$

  see the book page 371
- What happens as you change w? What should the phase plane look like? Check you work using pplane OR https://aeb019.hosted.uark.edu/pplane.html
- Are solutions going to end up at the fixed point? Are solutions going AWAY from the fixed point?

PART1. (solutions)

- $(0, 0)$
- $x(t) = Ae^{-t}$ and $y(t) = Be^{-kt}$
- $y(t) = B\left(e^{-t}\right)^k = B\left(\frac{x}{A}\right)^k = Cx^k$
  Here we rewrote the exponent and then solve the x equation for $e^{-t} = x/A$.
  This means that curves in the phase plan depend on that exponent k.
- When $k < 0$ solutions go toward the fixed point, otherwise they first go towards and then go away.

PART2. (solutions)

- $(0, 0)$
- $x(t) = A\cos(wt) + B\sin(wt)$ and $y(t) = -Aw\sin(wt) + Bw\cos(wt)$
- This means that curves in the phase plan depend on w this gives the shape of the ellipse.
- Trajectories circle the fixed point, but do not go towards or away from it.