Differential Equations - Homework 30

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Homework 30

Directions: Do the following book problems
NOTE: In the 3rd edition of the book this is chapter 5.4 - Multiple Eigenvalue Solutions and 5.6 - Nonhomogeneous Linear Systems

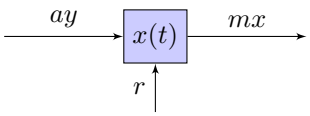
Section 5.5 problems 2, 4
Section 5.7 problems 2, 7

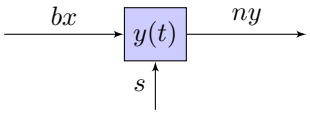
Directions: Do the following additional applied problems

Application Problem:

Lewis Fry Richardson was an English mathematician, physicist, meteorologist, psychologist and pacifist in the early 1900's. He developed his mathematical model for an arms race after he served in the medical corps during World War 1. Richardson was seeking a model that could predict large-scale military conflicts and assumed that the build of of arms would precede the large scale war. His model seeks to predict how two countries react to each others arms expenditures.

Consider two nations, where $x(t)$ and $y(t)$ represent each nations expenditures on arms at time t. The compartmental diagrams modeling this system are





Here a and b represent each nations response to the other nations spending on arms For example, for every unit of currency nation y spends on its arms supply, then nation x increases its arms spending by a. The parameters m and n represent each nations fatigue over their own spending, or their reluctance to spend more of their budget on arms. In other words would the country rather spend its money on non-military spending (food) or does it prioritize spending on military (arms). The parameters r and s represent what happens in the absence of the interaction, so if a, b, m, n are all zero then does the nation act in a peaceful way and disarm $r, s < 0$ or continue arming $r, s > 0$.

1. First write down the system of differential equations by reading from the compartmental diagram.
2. Now solve this system for the following parameters assuming initial conditions of $x(0) = 1$ and $y(0) = 1$ meaning that initially the two nations have equal spending

   - Match up 1 NATION 1 - reactionary and hostile

     $$a = 2, \quad m = 1, \quad r = 1$$

     NATION 2 - less reactionary, more likely to spend on food, and neither peaceful or hostile

     $$b = 1, \quad n = 3, \quad s = 0$$

   - Match up 2 - what if we change the interaction to: NATION 1 - reactionary and hostile

     $$a = 2, \quad m = 1, \quad r = 1$$

     NATION 2 - non-reactionary and non-hostile

     $$b = 0, \quad n = 1, \quad s = -1$$

3. Finally discuss your two different solutions in terms of the model. In each match up, what is the long term outlook for the arms race?

   NOTE: You could (optional) solve this system in general to see what kinds of solutions you might get. Are there any values where you get oscillating solutions and what does this mean physically? What are the conditions for disarmament?

Other Notes:

1. Come get help with eigenvalues and eigenvectors if you need it.