

Differential Equations – Homework 8

Professor:

Dr. Joanna Bieri

joanna_bieri@redlands.edu

Your goal today is to review First Order Equations. You should be able to solve all of these problems - in the past these were practice problems for Exam 1. Really test your understanding.

1. Show that the equation

$$(e^y + y \cos x)dx + (xe^y + \sin x)dy = 0$$

is exact. Solve the equation subject to the initial condition $y(0) = 0$.

2. The rate of change of alligator population P in a swamp is proportional to the square of P .

$$\frac{dP}{dt} = kP^2$$

The swamp contained a dozen alligators in 1988 and two dozen in 1998. When will there be four dozen alligators? Show all your work!

3. Using the substitution form $u = ax + by + c$, solve the differential equation

$$\frac{dy}{dx} = (x + y)^5 - 1.$$

4. Find the particular solution to the separable differential equation:

$$x^2 \frac{dy}{dx} = x^3 y^2 + y^2, \quad y(1) = 1.$$

5. Solve the following differential equations using the method of your choice. In each case show that your answer is correct.

$$3y + x^4 y' = 2xy.$$

$$e^x + ye^{xy} + (e^y + xe^{xy})y' = 0$$

$$3y + x^4 y' = 2xy$$

$$x^2 y' = xy + 3y^2$$

6. Use the method of integrating factor to solve for the general solution of

$$2x \frac{dy}{dx} + 4x^2 y = 2x^2.$$

7. Apply the existence and uniqueness theorem and say whether there are any (x_0, y_0) such that the following ODE's might have existence or uniqueness problems. THEN - plot a slope field (using dfield) with some typical solution curves that support your existence and uniqueness conclusions. Please discuss how the slope field supports your claims. (Like what the book does in figure 1.3.10)

$$\frac{dy}{dx} = \sqrt{x - y} \quad y(x_0) = y_0$$

$$y^2 \frac{dy}{dx} = x - 1 \quad y(x_0) = y_0$$

$$\frac{dy}{dx} = \ln(y^2 - 1) \quad y(x_0) = y_0$$