

Differential Equations – Introduction to Numerical Methods

Professor:

Dr. Joanna Bieri

joanna_bieri@redlands.edu

Understand Eulers Method

GOAL 1 - Explore Euler's Method first using a differential equation that we can solve:

$$\frac{dy}{dx} = x + y \quad y(0) = 1$$

Why do a numerical method on something we can solve? This will help us understand our method and check our work. After we test and better understand our method then we will have confidence in using it for a problem that we cannot solve

1. First CLASSIFY and SOLVE this ODE.
2. Write down the iterative formula for Euler's Method applied to this ODE.
3. Fill in the table with values, calculating the values by hand using the given h or step size values keep numbers out to the third decimal place and then use natural rounding.

x	Approx y with $h = 0.2$	Approx y with $h = 0.02$	Actual Value from the ODE solution
0			
0.2			
0.4			
0.6			
0.8			
1			

Is your answer an underestimate or over estimate?

4. Calculate the method error for each of your approximations above. We can only do this because we know the real answer!

x	Error y with $h = 0.2$	Error y with $h = 0.02$
0		
0.2		
0.4		
0.6		
0.8		
1		

5. Answer the following questions:
 - Plot the real solution (Python or Desmos) and then sketch on that solution what Euler's method is doing. Is your solution an under estimate or over estimate? Can you explain how under or over estimates might have to do with curvature of the real solution?
 - How did changing your step size h change your error? Was it better or worse with smaller step size?
 - Does your error increase or decrease as x gets larger?
6. Now that you have done this by hand, you can use a computer to simplify your work. Start by writing some PSEUDO CODE - what does the computer need to do:
 - What did you need to know to get started?
 - What were the steps you repeated over and over? How did you know to stop? (This is your "for" loop)
 - What do you want as your output?

You can try to write this code yourself, or use the link to the Colab notebook.

Use Eulers Method

GOAL 2 - Now we will use Euler's Method to find solutions to ODE's that we cannot solve by hand.

$$\frac{dy}{dx} = e^{x^2} \quad y(0) = 1$$

1. Make an argument as to whether or not you will expect your solution to be concave up or down? Will you expect the output of Euler's Method to be and under or over estimate?
2. Write down the iterative formula for Euler's Method applied to this ODE.

3. Use Python or a calculator to calculate $y(1)$ use the h values from the table below:

h	Euler's Method Solution $y(1)$
.1	
.01	
.001	
.0001	
...	

4. Answer the following questions:

- What do you notice about the solutions as you are decreasing your h values?
- What do you notice about the TIME that it takes to run a simulation for each of the h values?
- Can you make a prediction about how much error is in your solution based on the "convergence" of your solutions as h gets smaller? In other words, to how many decimal points do you trust your solution?

Repeat the steps 1-4 for a new problem!

$$y' = x^2 - y^2 \quad y(0) = 1$$

Use Python and Euler's Method to calculate $y(2)$ using the h values from the table below:

h	Euler's Method Solution $y(2)$
.1	
.01	
.001	
.0001	
...	