

Differential Equations – Introduction to Numerical Methods

Professor:

Dr. Joanna Bieri

joanna_bieri@redlands.edu

Improved Eulers Method

GOAL 1 - Explore Improved Euler's Method first using a differential equation that we can solve. This is the same one we solved before.

$$\frac{dy}{dx} = x + y \quad y(0) = 1$$

We always start with a problem we can solve so we can check our answers are reasonable.

1. Fill in the table with values, calculating the values by hand using the given h or step size values keep numbers out to the third decimal place and then use natural rounding.

x	Approx y with $h = 0.2$	Approx y with $h = 0.1$	Actual Value from the ODE solution
0			
0.2			
0.4			
0.6			
0.8			
1			

Is your answer an underestimate or over estimate?

2. Calculate the method error for each of your approximations above. We can only do this because we know the real answer!

x	Error y with $h = 0.2$	Error y with $h = 0.1$
0		
0.2		
0.4		
0.6		
0.8		
1		

3. Talk about the ratio of errors.

$$hvals = [.04, .02, .01, .005, .0025]$$

What do you expect for a second order method if we are dividing the step size in half?

Use Improved Euler's Method

GOAL 2 - Now we will use Improved Euler's Method to find solutions to ODE's that we cannot solve by hand.

$$\frac{dy}{dx} = e^{x^2} \quad y(0) = 1$$

1. Write down the iterative formula for Improved Euler's Method applied to this ODE.
2. Use Python to calculate $y(1)$ use the h values from the table below:

h	Im- proved Euler's Method Solution $y(1)$
.04	
.02	
.01	
.005	
...	

3. Answer the following questions:
 - What do you notice about the solutions as you are decreasing your h values?
 - Can you make a prediction about how much error is in your solution based on the "convergence" of your solutions as h gets smaller? In other words, to how many decimal points do you trust your solution?
4. Do an error analysis: ratio of errors for

$$hvals = [.04, .02, .01, .005, .0025]$$

Repeat the steps 1-4 for a new problem!

$$y' = x^2 - y^2 \quad y(0) = 1$$

Use Python and Improved Euler's Method to calculate $y(2)$ using the h values from the table below:

h	Improv'd Euler's Method Solution $y(2)$
.04	
.02	
.01	
.005	
...	