Differential Equations - In Class Activity

Professor:
Dr. Joanna Bieri
joanna_bieri@redlands.edu

The Zombie Apocalypse Hits Redlands!!!

Imagine that exactly 999,990 people (men, women, children) live in Redlands (just outside our classroom) at this very minute. Today, 10 zombies rose from the grave and began attacking people. Any person who has their brain eaten by a zombie also becomes a zombie. The zombies are restricted to the city of Redlands, the government has set up an impenetrable quarantine. During the initial panic and confusion, when the brain resources are plentiful, we notice that each zombie will convert about two people into new zombies each day. As math geniuses we lock ourselves in the tower of Appleton hall, with its super secret, city wide, zombie surveillance room, and decide to model this outbreak using differential equations, for the good of the future of humanity!

GOAL: Write down equations that model the number of zombies. Then solve these equations to obtain predictions for the number of zombies after t days. Let $z(t)$ be the number of zombies after t days.

1. In our first few days of the outbreak we see that at $t = 0$ we have $z = 10$ and at $t = 1$ we have $z = 30$ zombies. There seems to be no scarcity of resources, the zombies are eating as much as they like. Using this limited information we draw a compartmental diagram and write down a simple (non-logistic) differential equation along with initial conditions.

- Classify and solve this ODE.
- Find the growth rate r.
- What happens as $t \to \infty$?
- Is this realistic?
- Open dfield and plot the ODE for a variety of initial conditions. Is there any hope for us?
- With $z(0) = 10$ about how long until 50% of the population is zombies? Could there have been an initial number of zombies that we could have overcome, based on our assumptions?

2. As the days march on and Redlands sinks further into devastation, we notice that as the number of people decreases it becomes more difficult for the zombies to find them. The zombies are exhausting their human food resource and their reproduction seems to slow. People are a limited resource meaning that the zombies cannot grow infinitely. Well this changes everything and we must go back to the drawing board to re-evaluate our model. So we make the broad assumption that the growth rate decreases linearly with the increase of zombies, so initially, when $z = 10$, the growth rate was a maximum (this is the maximum growth rate we found in part 1) and once everyone is a zombie, $z = 100,000$, the growth rate must be zero.

- Draw a graph of the growth rate as a function of the number of zombies, assume a linear relationship, with your growth rate r from part 1. as your maximum growth rate.
- Find the equation for the line in your graph.
- Draw a compartmental diagram for the zombie growth, but now use this new functional growth rate.
- Write down the differential equation and solve it subject to the appropriate initial conditions.
- Now what happens as $t \to \infty$. Is this more or less realistic than our first model?
- Open dfield and plot the ODE for a variety of initial condition. Is there any hope for us?
- With $z(0) = 10$, about how long until 50% of the population is zombies? Does our outlook seem better or worse now?

3. (EXTRA) If you have time, what would happen if the zombies were more aggressive, they tried to infect more people per day, or the initial number of zombies were different. Playing with parameters in your model is a good way to better understand the system! You can either use a plot of your solution or use dfield to approach this problem. What parts of your model would change?

4. (EXTRA) What if we started helping the citizens of Redlands by throwing the computers from our helicopter, yeah we have a helicopter, onto the Zombies heads? Say we were able to kill Z_{kill} Zombies each day. Can you draw a compartmental diagram for this case. What would the differential equation for this system be? Plot this ODE on dfield, what is the minimum number zombies that we need to kill per day to save Redlands with in the first week?

5. (EXTRA) What if the zombie growth rate was not a linear function of z but a nonlinear function because the remaining humans were the strong, smart, best of the best. What function might you pick? Draw a graph. How does this change the ODE model?