The Improved Euler Method.

As we saw the Euler Method

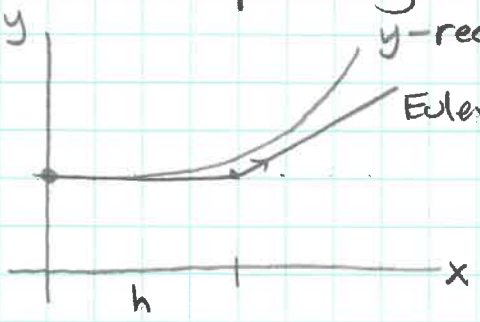
$$y_{n+1} = y_n + h f(x_n, y_n)$$

has error $E \propto h$ or is an order one method.

How can we develop higher order (More Accurate) methods?

- Better approximations for the slope
- Better approximations for the derivative.

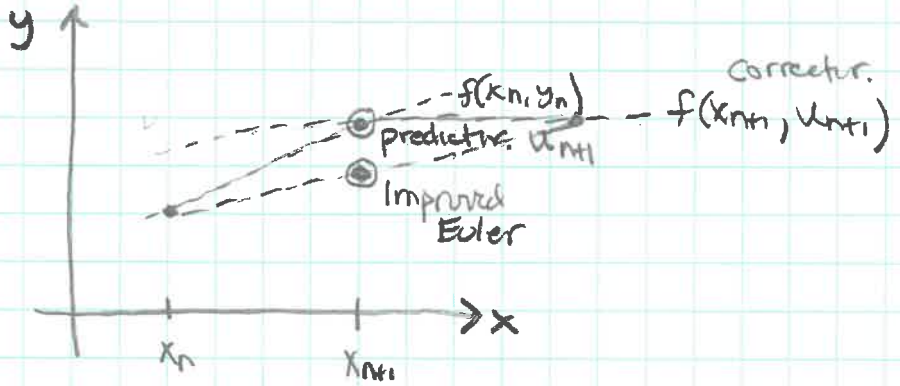
Consider — the error in Euler comes from predicting the slope using the left side point.



we choose the slope at x_n, y_n ... but what if we could get more information.

Predictor - Corrector Methods — Improved Euler

$$y' = f(x,y)$$
$$y(x_0) = y_0$$



1. Do an euler step to get the new predicted value

$$u_{n+1} = y_n + hk_1$$

where $k_1 = f(x_n, y_n)$

2. Find the slope at the predictor point.

$$k_2 = f(x_n, u_{n+1})$$

3. Correct the euler step using the average slope

$$y_{n+1} = y_n + \frac{h}{2}(k_1 + k_2)$$

This method has error $E \propto h^2$ it is order two

BUT there are twice as many computations.