Lecture 14 Differential Eqns.

- Hand in HW 10
- HW 11 due Fri
- HW 12 due Mon.
- Exam 2 Friday 2/25

– General Soln Method → Linear Eqns w/ Constant Coef.

1. Classify the ODE $Ly = f(x)$
   - Linear constant coef.
   - Order:
   - Homogeneous?:

2. Solve $Ly_H = 0$ to find the homogeneous soln.
   A. Write down Characteristic Eqn. (CE)
   B. Find and Classify roots of CE
   C. Build solution from roots.
   - REAL DISTINCT: $c_1 e^{r_1 x} + c_2 e^{r_2 x} + \cdots$
   - REAL REPEATED: multiplicity m $(c_1 + c_2 x + \cdots + c_m x^{m-1}) e^{r_1 x}$
   - COMPLEX ROOTS: $r = a \pm bi$ $e^{ax}(c_1 \sin bx + c_2 \cos bx)$

3. Solve $Ly_{NH} = f(x)$ to find nonhomogeneous soln.
   A. Look ONLY at $f(x)$ and guess that y_{NH} has the same form.
   B. Compare your guess with y_H – make sure there are no duplications.
   C. If duplication mult through by x^s until no duplication exists.
   D. Plug your solution into Ly_{NH} to solve for the coef.

GENERAL GUESS: $y_{NH} = x^s(c_0 + c_1 x + c_2 x^2 + \ldots + c_n x^n) e^{ax}(A\sin(bx) + B\cos(bx))$

4. Write down the GENERAL SOLN: $y = y_H + y_{NH}$

5. Apply Initial Conditions.

More Examples ...

FROM LAST TIME: $Ly = x^2e^{2x} + x\sin 3x$

Characteristic Eqn has roots $(r-2)^3(r^2+9)=0$

$$y_H = (c_1 * c_2x + c_3x^2)e^{2x} + c_4\sin 3x + c_5\cos 3x$$

Guess: $y_{NH} = \underline{(c_1+c_2x+c_3x^2)e^{2x}} + \underline{(c_4+c_5x)}\underline{(c_6\sin 3x + c_7\cos 3x)}$

Look @ $y_H \rightarrow$ there are LOTS of duplications!!

Better Guess:

$$y_{NH} = x^3(c_1+c_2x+c_3x^2)e^{2x} + x(c_4+c_5x)(c_6\sin 3x + c_7\cos 3x)$$

mult by x^3 to get rid of duplication in 1st term

mult by x to get rid of duplication in 2nd term.

Ex

assume $y_H = (c_1+c_2x)e^{2x}$

and $f(x) = x\sin x + \cos x$

What is a good guess?

Ex

assume $y_H = c_1e^{2x} + e^{3x}(c_2\sin x + c_3\cos x)$

and $f(x) = e^{2x} + \sin x + \sin 2x$

What is a good guess?

Ex

assume $y_H = c_1 + c_2x + c_3x^2 + c_4e^{4x}$

and $f(x) = x^3 + 7e^{2x}$

What is a good guess?

EX Solve the ODE $y'' + y = 4x\sin x + \cos 2x$

NOTE... there is some annoying algebra here... be careful in your calculations it will pay off!

$$y_H = c_1 \sin x + c_2 \cos x$$

guess $y_{NH} = (c_1 x + c_2 x^2)\sin x + (c_3 x + c_4 x^2)\cos x + c_5 \sin 2x + c_6 \cos 2x$

$$c_1 = 1 \quad c_2 = 0 \quad c_3 = 0 \quad c_4 = -1$$
$$c_5 = 0 \quad c_6 = -1/3$$

$$y_{NH} = x\sin x - x^2\cos x - 1/3 \cos 2x$$

$$y = c_1 \sin x + c_2 \cos x + x\sin x - x^2 \cos x - 1/3 \cos 2x.$$

EX Solve the ODE $y^{(3)} - 6y'' + 9y' = 5 + \sin x$

$$y_H = c_1 + c_2 e^{3x} + c_3 x e^{3x}$$

guess $y_{NH} = Ax + B\sin x + C\cos x$

$$A = 5/9 \quad B = .06 \quad C = -.08$$

$$y_{NH} = 5/9\, x + .06 \sin x - .08 \cos x$$

$$y = c_1 + c_2 e^{3x} + c_3 x e^{3x} + 5/9\, x + \underset{\frac{3}{50}}{.06}\sin x - \underset{\frac{2}{25}}{.08}\cos x.$$