Partial Differential Equations - Homework Day 8

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Read and Take Notes

1. *Farlow* - Lesson 9 - We are starting Eignefunction Expansions for Non Homogeneous Equations.

Homework

1. *Homework Problems*

   1. Laplace's Equation on the unit square, centered at origin, with one non-homogeneous boundary condition:

$$\begin{aligned}
&\nabla^2 u = 0\\
&u(-1, y) = 0\\
&u(1, y) = 0\\
&u(x, -1) = 0\\
&u(x, 1) = \sin(\pi x)
\end{aligned}$$

   please show all the steps! Explain in words what you are doing with each step.

Extra Problems Just for Fun! Totally Optional - but if you want to show off, do them. :)

You do not need to hand these in as homework.

E1. Laplace's Equation on the rectangle with a flux: (note: this square is not centered at the origin)

$$\begin{aligned}
&\nabla^2 u = 0\\
&u_x(0, y) = 0\\
&u_x(L, y) = 0\\
&u(x, 0) = 0\\
&u(x, H) = f(x)
\end{aligned}$$

E2. What is the solution to the completely homogeneous Laplace's Equation:

$$\begin{aligned}
&\nabla^2 u = 0\\
&u(-1, y) = 0\\
&u(1, y) = 0\\
&u(x, -1) = 0\\
&u(x, 1) = 0
\end{aligned}$$

You can use your work from problem 1. Does the solution make physical sense? Why?

E3. Solve Laplace's Equation inside the **semicircle**

$$0 < r < a \quad \text{and} \quad 0 < \theta < \pi$$

where the base of the semicircle ($\theta = 0$ and $\theta = \pi$) is kept at zero temperature and the top arch ($r = a$) is at a prescribed temperature. Remember to use the polar coordinate form of the Lapacian ∇^2.

$$\begin{aligned}
&\nabla^2 u = 0\\
&u(a, \theta) = g(\theta) = 1\\
&u(r, 0) = 0\\
&u(r, \pi) = 0
\end{aligned}$$

What is the additional condition that we require for $u(r, \theta)$ when $r = 0$?

NOTE: General Solution: $u(r, \theta) = \sum_{n=1}^{\infty} B_n r^n \sin(n\theta)$