Farlow Lesson 1.

ODE - ordinary differential equation → unknown function depends on one variable.

PDE - partial differential equation → unknown function depends on more than one variable.

Notation: $u_t = \frac{\partial u}{\partial t}$ $u_x = \frac{\partial u}{\partial x}$ $u_{xx} = \frac{\partial^2 u}{\partial x^2}$

WELL KNOWN PDES...

$u_t = u_{xx}$ one dimensional heat equation

$u_t = u_{xx} + u_{yy}$ two dimensional heat equation

$u_{rr} + \frac{1}{r} u_r + \frac{1}{r^2} u_{\theta\theta} = 0$ Laplace's equation in polar coordinates.

$u_{tt} = u_{xx} + u_{yy} + u_{zz}$ ware equation in three dimensions

$u_{tt} = u_{xx} + \alpha u_t + \beta u$ telegraph equation.

TEN SOLUTION METHODS

1. Separation of variables - reduces a pde in n variables in to n ode's
2. integral transforms - reduces a pde in n variables to one in n-1 variables. So a pde in two variables can be reduced to an ode.
3. Change of coordinates - Changes a pde into an ode or a simpler pde by changing the coordinate system.

4. Transformation of the dependent variable – transforms the unknown of the PDE into a new unknown that is easier to find.
5. Numerical Methods – Change a PDE into a system of recurrence relations that can be solved by a computer.
6. Pertubation Methods – changes a nonlinear problem into a sequence of linear ones that approximate the nonlinear one
7. Impulse-response Technique – Decomposes initial and boundary conditions of the problem into simple impulses and finds the response to each impulse.
8. Integral equations – changes a PDE into an integral equation (where the unknown is inside the integral)
9. Calculus of Variations – Find the solution to PDEs by reformulating the equation as a minimization problem, where the minimum of a certain expression is the solution to the PDE
10. Eigenfunction Expansion – attempts to find the solution of a PDE as an infinite sum of eigenfunctions.

KINDS OF PDES (CLASSIFICATION)

1. Order of the PDE = the order of the highest partial derivative.
2. Number of Variables = the # of independent variables
3. Linearity – linear vs nonlinear forms of the dependent variable.

Ex second Order Linear Eqn in Two Variables...

$$Au_{xx} + Bu_{xy} + Cu_{yy} + Du_x + Eu_y + Fu = G$$

A, B, C, D, E, F and G are either constants or functions of the independent variable

4. Homogeneity – a pde is homogeneous if $G(x,y) = 0$ and non-homogeneous otherwise

5. Kinds of Coefficients – constant vs variable coefficients

6. Three Basic Types of Linear Equations

PARABOLIC – $B^2 - 4AC = 0$
used to describe heat flow and diffusion processes

HYPERBOLIC – $B^2 - 4AC > 0$
describe vibrating systems and wave motion.

ELLIPTIC – $B^2 - 4AC < 0$
steady-state phenomena.

Ex $u_t = u_{xx}$
second order, linear, homogeneous, two variables constant coefficients.
$A = 1 \quad B = 0 \quad C = 0 \quad B^2 - 4AC = 0$ parabolic.

Ex $u_{yx} = 0 \quad A = 0 \quad B = 1 \quad C = 0 \quad B^2 - 4AC = 1$ hyperbolic
second order, linear homogeneous, two variables constant coefficients.